The Schrödinger equation is the fundamental governing equation of non-relativistic quantum mechanics. It plays the same role in the quantum world that Newton’s second law (F = ma) plays in classical mechanics. Instead of predicting a precise trajectory, it calculates the wavefunction, which contains all the statistical information about a system.

Here is the mathematical breakdown of its two main forms.

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1. The Time-Dependent Schrödinger Equation (TDSE)

This is the most general form of the equation. It describes how a quantum system evolves over time.

$$i\hbar \frac{\partial}{\partial t}\Psi(\mathbf{r}, t) = \hat{H}\Psi(\mathbf{r}, t)$$

Breaking Down the Components:

i: The imaginary unit (√(-1)). Its presence ensures that quantum states wave and oscillate rather than simply decaying like heat.

hbar: The reduced Planck constant (hbar = (h)/(2π)), which sets the scale of the quantum world.

(∂)/(∂ t): The partial derivative with respect to time, representing how the system changes second by second.

Ψ(r, t): The wavefunction. It is a function of position r and time t. The absolute square $|\Psi|^2$ gives the probability density of finding a particle at a specific place and time.

hatH: The Hamiltonian operator. In quantum mechanics, physical observables are represented by operators. The Hamiltonian represents the total energy of the system.

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2. The Hamiltonian Operator in Detail

To do actual calculations for a particle moving in space, we must expand the Hamiltonian operator hatH into its kinetic and potential energy components:

$$\hat{H} = \hat{T} + \hat{V}$$

Where:

Kinetic Energy Operator (hatT): Derived from momentum, it is expressed as -(hbar^2)/(2m)∇^2, where m is the mass and ∇^2 (del-squared) is the Laplacian operator checking spatial curvature.

Potential Energy Operator (hatV): Represented by V(r, t), which depends on the environment (e.g., an electric field or a gravitational well).

Putting it all together in 3D space:

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t)\right]\Psi(\mathbf{r},t)$$

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3. The Time-Independent Schrödinger Equation (TISE)

If the potential energy V does not depend on time (V(r, t) = V(r)), we can use a mathematical technique called separation of variables. We split the wavefunction into a spatial part and a temporal part: Ψ(r, t) = ψ(r) e$^{(}$-iEt/hbar$^{)}$.

Plugging this back in yields the Time-Independent Schrödinger Equation:

$$\hat{H}\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

Or explicitly:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

Mathematical Significance of the TISE:

An Eigenvalue Problem: Mathematically, this is an eigenvalue equation. hatH is a linear operator, ψ(r) is the eigenfunction, and E is the eigenvalue.

Quantization: Solving this equation under specific boundary conditions restricts the allowable values of E. This is precisely where "quantization" comes from in quantum mechanics—energy levels cannot be just anything; they must be specific discrete values.

Summary of Mathematical Interpretations

Concept	Mathematical Representation	Meaning
Probability Conservation	$\int{-\infty}^{\infty} | \Psi(\mathbf{r}, t)	
Linearity	If ψ_1 and ψ_2 are solutions, then $c_1\psi_1 + c_2\psi_2$ is also a solution.	Explains quantum superposition.
Hermitian Operator	hatH = hatH^†	Guarantees that the energy eigenvalues (E) will always be real numbers, which can be measured in a lab.

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