Introduction

Quantum electrodynamics, usually abbreviated QED, is the quantum theory of electrically charged matter interacting with light. In its most familiar form, the charged matter is the electron-positron field, and light is the photon field. The central physical question is simple to state:

> What are the laws governing electrons, positrons, and photons when quantum mechanics, special relativity, and electromagnetism must all be true at once?

That question is simple, but its answer is one of the great achievements of twentieth-century physics. QED explains why atoms have the spectra they do, why electrons scatter from other charged particles with specific angular distributions, why the electron magnetic moment differs slightly from the value predicted by the simplest Dirac theory, and why empty space can influence electromagnetic propagation through quantum fluctuations. The relativistic theory of the electron began with Dirac's 1928 equation, which correctly incorporated spin and led naturally to the existence of antiparticles (Dirac 1928). The modern covariant perturbative formulation of QED was developed through the work of Tomonaga, Schwinger, Feynman, and Dyson in the 1940s (Tomonaga 1946; Schwinger 1948a; Feynman 1949; Dyson 1949).

This book is a graduate path through that theory. It is not only about learning a set of Feynman rules. It is about understanding why those rules have the form they do, what assumptions make them reliable, how infinities are handled without losing predictivity, and how precision measurements become tests of quantum field theory itself.

Why QED is not just "quantum mechanics plus electromagnetism"

At first sight, QED might sound like ordinary quantum mechanics applied to electromagnetic forces. In nonrelativistic quantum mechanics, one may study an electron in a Coulomb potential,

$$V(r) = -\frac{e^2}{4\pi r},$$

and obtain the leading structure of hydrogen. This is already a powerful approximation. But it is not yet QED.

The difference is that QED treats both matter and radiation as fields. A field is a physical quantity assigned to every point of spacetime. For example, the classical electromagnetic field assigns electric and magnetic field values to spacetime events. In quantum field theory, fields become operator-valued objects or, in the path-integral language, integration variables whose fluctuations determine quantum amplitudes. The particle picture then emerges from the field picture: photons are quanta of the electromagnetic field, and electrons and positrons are quanta of the Dirac field.

This shift is not optional. Special relativity and quantum mechanics together make fixed-particle-number quantum mechanics inadequate at sufficiently high energies. If enough energy is available, particles can be created. A photon can produce an electron-positron pair in the presence of another charged object; an electron and positron can annihilate into photons. A theory with a fixed number of particles cannot describe such processes as fundamental events. A quantum field theory can.

A useful first example is electron-positron annihilation:

$$e^- + e^+ \to \gamma + \gamma.$$

In a fixed-particle-number theory, the initial state has two massive charged particles and the final state has two massless neutral particles. The identity and number of particles have changed. QED describes this naturally because the electron, positron, and photon are excitations of underlying fields, and the interaction allows energy, momentum, angular momentum, and charge to be redistributed among those excitations.

The basic object: the QED Lagrangian

A large part of this book will revolve around one compact formula, the QED Lagrangian density,

$$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi.$$

Do not worry if every symbol is not yet familiar. The early chapters build each piece from first principles. For orientation, here is the meaning.

The symbol ψ denotes the Dirac field, the relativistic quantum field whose quanta are electrons and positrons. The symbol barψ is its Dirac adjoint, a conjugate object needed to build Lorentz-invariant quantities. The matrices γ^μ are gamma matrices, which encode the spinorial structure required by special relativity. The parameter m is the electron mass.

The symbol A_μ denotes the electromagnetic four-potential. From it one constructs the electromagnetic field-strength tensor,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

This tensor contains the electric and magnetic fields in relativistic form. The derivative

$$D_\mu = \partial_\mu + ieA_\mu$$

is called the gauge-covariant derivative. It is the derivative that knows about electromagnetic interaction. The constant e is the electric charge coupling.

The reason this Lagrangian is so important is not merely that it gives the correct equations. It expresses the principles that organize the theory: locality, Lorentz invariance, gauge symmetry, and quantum mechanics. A local theory is one whose interactions occur at the same spacetime point in the Lagrangian. A Lorentz-invariant theory has laws that take the same form in all inertial frames. A gauge symmetry is a redundancy in description: different mathematical potentials A_μ may describe the same physical electromagnetic fields. QED is built so that physical predictions do not depend on that redundancy.

For example, changing the electromagnetic potential by

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \alpha(x)$$

does not change $F_{(\mu nu)}$. If the Dirac field is transformed at the same time by a local phase rotation,

$$\psi(x) \to e^{-ie\alpha(x)}\psi(x),$$

the QED Lagrangian keeps the same physical content. This is local U(1) gauge invariance. Here U(1) means the group of complex phase rotations of unit magnitude. This symmetry is not decorative. It controls the allowed interaction, protects charge conservation, and leads to powerful identities among scattering amplitudes.

The empirical seriousness of QED

QED is not admired only for elegance. It is admired because it makes extraordinarily precise predictions. One famous example is the electron's magnetic moment. In the simplest Dirac theory, the electron has a gyromagnetic factor g=2. QED predicts radiative corrections, meaning corrections produced by virtual photon and electron-positron fluctuations. Schwinger's one-loop result gives the leading anomalous contribution,

$$a_e \equiv \frac{g-2}{2} = \frac{\alpha}{2\pi} + \cdots,$$

where α is the fine-structure constant (Schwinger 1948b). Modern calculations include much higher orders, and comparison with precision measurements of the electron magnetic moment is one of the sharpest tests of QED (Hanneke, Fogwell, and Gabrielse 2008; Aoyama et al. 2012).

Another landmark is the Lamb shift, a small splitting in hydrogen energy levels that are degenerate in the simplest Dirac-Coulomb treatment. Lamb and Retherford measured this splitting using microwave methods (Lamb and Retherford 1947), and its explanation became one of the early triumphs of renormalized QED. The Lamb shift teaches an important lesson: even when no real photons are emitted or absorbed, quantum fluctuations of the electromagnetic field affect observable atomic energies.

These examples also warn us against a common misunderstanding. QED is not a theory in which "virtual particles are tiny classical objects briefly appearing and disappearing." Virtual particles are internal lines in a perturbative expansion of quantum amplitudes. They are useful bookkeeping devices in Feynman diagrams, not directly observed particles with classical trajectories. Their effects, however, contribute to measurable quantities such as scattering cross sections, magnetic moments, and energy shifts.

The language of amplitudes

A major goal of this book is to teach you how QED predicts scattering and decay probabilities. The central object is the amplitude. An amplitude is a complex number associated with a process. Probabilities and cross sections are obtained from absolute squares of amplitudes, after including phase space and experimental conditions.

For example, in electron-muon scattering,

$$e^- + \mu^- \to e^- + \mu^-,$$

the leading QED process is mediated by photon exchange. A Feynman diagram represents this compactly: an electron line and a muon line exchange a photon. But the diagram is not a picture of a little photon traveling along a definite path between two particles. It is a graphical representation of a mathematical term in a perturbation series.

The perturbation series is an expansion in the electromagnetic coupling. In natural units, the relevant dimensionless parameter is the fine-structure constant,

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}.$$

Because α is small at ordinary laboratory energies, many QED processes are well described by the first few terms. A tree-level calculation is the leading calculation with no closed loops in the Feynman diagram. A loop calculation includes internal momentum integrations and accounts for quantum fluctuations beyond the leading approximation. Loops are essential for precision physics, but they bring new mathematical issues, especially divergences.

A divergence is an expression that becomes infinite in an intermediate calculation. In QED, ultraviolet divergences appear when loop momenta become arbitrarily large. The modern treatment has two steps. First, one introduces a regularization, a mathematically controlled way to make divergent expressions temporarily finite. Second, one performs renormalization, rewriting the theory in terms of physical parameters such as the observed electron mass and charge. Renormalization does not mean hiding infinities by hand; it is a systematic relation between the parameters in the Lagrangian and measured quantities. Dyson's work was central in showing the equivalence and systematic structure of the covariant perturbation theories developed by Tomonaga, Schwinger, and Feynman (Dyson 1949).

What "gauge invariance" will do for us

Gauge invariance is one of the main characters of this book. At first, it may feel like a technical complication: we introduce a potential A_μ, then admit that many choices of A_μ describe the same physical electromagnetic fields, then must fix a gauge to compute. But gauge symmetry is not merely a nuisance. It is a guide.

A simple example comes from photon polarization. A real photon has two physical transverse polarization states. But the four-vector potential $A\mu$ has four components. Gauge redundancy and constraints remove the unphysical components. If a calculation predicts a physical answer that depends on an unphysical photon polarization, something has gone wrong. Ward identities make this statement precise. They are relations among amplitudes that follow from gauge invariance. In practical calculations, they are among the best error checks available.

For instance, in many QED amplitudes involving an external photon with polarization vector ε^μ, gauge invariance implies that replacing ε^μ by the photon momentum k^μ should give zero for the physical amplitude. This is not a random algebraic trick; it is the computational shadow of the fact that unphysical gauge degrees of freedom cannot affect observables.

The architecture of the book

The chapters ahead follow a deliberate progression.

We begin with the physical problem of light and matter. Before writing down formal machinery, we ask what must be explained: charged particles, radiation, scattering, spectra, antiparticles, conservation laws, and measurement. This establishes the need for a relativistic quantum field theory rather than a nonrelativistic wave mechanics with electromagnetic corrections.

We then review relativistic quantum mechanics and its limits. The Klein-Gordon and Dirac equations are essential stepping stones. They reveal both progress and tension: Lorentz covariance appears, spin emerges naturally in the Dirac theory, and negative-energy solutions lead toward antiparticles. But the fixed-particle interpretation breaks down, pushing us toward fields.

Next, we develop classical fields, symmetries, and Noether's theorem. Noether's theorem connects continuous symmetries with conserved quantities. Time-translation symmetry gives energy conservation, space-translation symmetry gives momentum conservation, rotational symmetry gives angular momentum conservation, and global phase symmetry gives charge conservation. This is the structural language in which QED is written.

The book then quantizes free fields: scalar fields, Dirac fields, and the electromagnetic field. Here we meet Fock space, creation and annihilation operators, propagators, and microcausality. Microcausality means that observables at spacelike-separated points must not influence one another, preserving compatibility with special relativity.

Several chapters focus on the Dirac and electromagnetic fields in detail. This is where spinor technology becomes practical. You will learn gamma-matrix algebra, spin sums, helicity, chirality, polarization vectors, and gauge fixing. These tools are not optional decoration; they are the grammar of QED calculation.

After that, we build the QED Lagrangian from local U(1) gauge invariance. The interaction between electrons and photons will no longer look like an arbitrary addition. It will appear as the natural consequence of demanding that local phase choices in the charged field have no physical effect.

We then turn to perturbation theory and the S-matrix. The S-matrix is the object that maps incoming asymptotic particle states to outgoing asymptotic particle states. In ordinary language, it is the mathematical device that lets us compute scattering and decay. Wick's theorem, time ordering, and connected diagrams will become the bridge from the Lagrangian to Feynman rules.

The middle chapters compute tree-level processes: electron-muon scattering, Bhabha scattering, Møller scattering, Compton scattering, annihilation, and pair production. These examples teach the workflow: identify external states, draw diagrams, write amplitudes, square them, sum and average over spins, integrate over phase space, and interpret the result.

The later chapters move into loops, renormalization, and the renormalization group. We study vacuum polarization, the electron self-energy, and the vertex correction. These are the basic one-loop structures from which much of QED's precision physics grows. The renormalization group then explains why the effective electromagnetic coupling depends on scale and why QED is best understood as part of a larger effective-field-theory framework at very high energies.

Finally, we study infrared physics, bound states, precision tests, discrete symmetries, external fields, anomalies, and effective field theory. These topics show that QED is not just a collection of scattering calculations. It is a flexible framework connecting atomic physics, high-energy scattering, precision metrology, and modern quantum field theory.

How to think while learning QED

QED rewards two habits: physical interpretation and algebraic discipline.

Physical interpretation means constantly asking what a formula is saying about measurable quantities. If you compute an amplitude, ask what experiment could probe it. If you choose a gauge, ask why the final answer must not depend on that choice. If a loop integral diverges, ask which parameter of the physical theory is being related to measurement.

Algebraic discipline means respecting signs, factors of i, spinor order, metric conventions, normalization conventions, and integration measures. Many QED errors are not conceptual; they are bookkeeping errors. But bookkeeping errors in quantum field theory can mimic conceptual problems. This book will therefore emphasize checks: dimensional analysis, Ward identities, limiting cases, crossing symmetry, and comparison with known results.

A useful example is Compton scattering,

$$e^- + \gamma \to e^- + \gamma .$$

At tree level there are two diagrams, not one. If you keep only one, the result fails gauge-invariance checks. The need for both diagrams is not an arbitrary rule; it reflects the structure of the interaction and the indistinguishability of allowed time orderings in the covariant amplitude. A good QED calculation is not just a sequence of manipulations. It is a structure in which symmetry, causality, and probability fit together.

What this book assumes and what it builds

This is a graduate-level book. It assumes familiarity with undergraduate quantum mechanics, special relativity, classical electromagnetism, and some mathematical methods such as Fourier transforms and complex integration. Prior exposure to Lagrangian mechanics and group theory will help, but the essential field-theoretic ideas are developed as they are needed.

The book builds toward research-level competence in standard QED calculations. By the end, you should be able to read and perform tree-level calculations confidently, understand the origin and handling of one-loop divergences, explain the meaning of renormalization conditions, use gauge invariance as a practical constraint, and recognize how QED fits into the broader structure of the Standard Model and effective field theory.

The deeper goal is not memorization. It is fluency. Fluency means that when you see

$$\bar{u}(p')(-ie\gamma^\mu)u(p)\frac{-ig_{\mu\nu}}{q^2+i\varepsilon}\bar{u}(k')(-ie\gamma^\nu)u(k),$$

you do not see a meaningless string of symbols. You see charged fermion currents exchanging a photon. You see Lorentz indices being contracted. You see a propagator carrying momentum transfer. You see where gauge choice may enter and why the physical answer must be gauge independent. You see the beginning of a cross-section calculation.

That is the kind of understanding this book aims to develop: mathematical enough to compute, physical enough to interpret, and careful enough to trust.

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Document information

Introduction

Project	Quantum Electrodynamics
Document	Document 1.4
Author	terry.mart
Verifier	Not verified
Downloaded	July 05, 2026 19:15 KST
Status	Working
Document link	https://www.theorytrace.com/projects/quantum-electrodynamics/documents/introduction/