

## Chapter 4: Quantizing Free Fields

Classical fields, as introduced in the previous chapter, are functions on spacetime. A scalar field  $\phi(x)$ , a spinor field  $\psi(x)$ , and an electromagnetic potential  $A_\mu(x)$  assign mathematical objects to each spacetime point  $x=(t,x)$ . Their equations of motion follow from an action, and their symmetries lead to conserved quantities.

Quantum electrodynamics requires one more step: these fields must become operators.

This is not a decorative change of notation. It changes the physical meaning of the theory. A classical field configuration describes a definite value of the field everywhere. A quantum field operator acts on a Hilbert space of states, and its excitations behave as particles. In this chapter we build that Hilbert space for free fields: scalar fields, Dirac fields, and photon fields.

The word free means that the fields do not yet interact with each other. A free electron field describes electrons and positrons that propagate without emitting or absorbing photons. A free electromagnetic field describes photons that propagate without interacting with charges. Real QED is interacting, but perturbation theory begins by understanding the free theory exactly and then treating interactions as controlled corrections.

Historically, this step was one of the decisive conceptual transitions from relativistic wave mechanics to quantum field theory. Dirac's early quantum theory of radiation already treated emission and absorption as changes in photon number (Dirac 1927). Pauli and Weisskopf later clarified that the Klein-Gordon equation should be understood as a field equation whose quantized excitations include particles and antiparticles, not as an ordinary one-particle probability wave equation (Pauli and Weisskopf 1934). Modern canonical quantization is now a standard route into quantum field theory, as presented for example in Peskin and Schroeder (1995) and Weinberg (1995).

Throughout this chapter we use natural units,

$$\hbar=c=1,$$

and the metric convention

$$\eta_{\mu\nu}=\text{diag}(1,-1,-1,-1).$$

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## 4.1 From classical coordinates to quantum operators

The simplest quantum system is not a field but a single particle moving in one dimension. Classically it has a coordinate  $q(t)$ , momentum  $p(t)$ , and Hamiltonian  $H(q,p)$ . Canonical quantization promotes  $q$  and  $p$  to operators satisfying

$$[\hat{q}, \hat{p}] = i.$$

The bracket

$$[A, B] \equiv AB - BA$$

is called a commutator. It measures the failure of two operators to commute. If two observables do not commute, they cannot generally be assigned sharp values simultaneously in one quantum state.

A classical field is like a mechanical system with infinitely many coordinates: one coordinate for each point  $x$  in space. For a real scalar field  $\varphi(t, x)$ , the canonical momentum is

$$\pi(t, x) = (\partial \mathcal{L}) / (\partial \dot{\varphi}(t, x)).$$

Canonical quantization promotes

$$\varphi(t, x), \pi(t, x)$$

to operators and imposes the equal-time commutation relations

$$[\varphi(t, x), \pi(t, y)] = i\delta^3(x - y),$$

$$[\varphi(t, x), \varphi(t, y)] = 0, [\pi(t, x), \pi(t, y)] = 0.$$

The delta function appears because each spatial point behaves like a separate degree of freedom. The phrase equal-time means that the commutator is imposed on a fixed time slice. Relativistic covariance is not obvious in this form, but it will reappear through the field equations and through microcausality.

A useful picture is this:

> A free quantum field is an infinite collection of coupled harmonic oscillators, one oscillator for each momentum mode.

The quantum of excitation of one such oscillator is what we call a particle.

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## 4.2 The real scalar field

The simplest relativistic field is a real scalar field. "Scalar" means that the field has no spin indices and is unchanged by Lorentz transformations except for the transformation of its spacetime argument. "Real" means

$$\varphi^*(x) = \varphi(x).$$

The free real scalar Lagrangian density is

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2.$$

The Euler-Lagrange equation gives the Klein-Gordon equation,

$$(\Box + m^2)\varphi = 0, \quad \text{Boxequiv } \partial_\mu \partial^\mu = (\partial_t)^2 - \nabla^2.$$

The canonical momentum is

$$\pi = \dot{\varphi}.$$

The Hamiltonian is

$$H = \int d^3x \left[ \frac{1}{2} \pi^2 + (\nabla \varphi)^2 + m^2 \varphi^2 \right].$$

Classically this Hamiltonian is positive for  $m^2 \geq 0$ . Quantum mechanically, we now express the field in modes.

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## 4.3 Mode expansion and creation operators

A plane-wave solution of the Klein-Gordon equation has the form

$$e^{-ip \cdot x}, \quad p \cdot x = p_\mu x^\mu = Et - \mathbf{p} \cdot \mathbf{x},$$

with the on-shell condition

$$p^0 = m, \quad E = \sqrt{p^2 + m^2}.$$

The quantized real scalar field is expanded as

$$\varphi(x) = \int (d^3p) \frac{1}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[ a(\mathbf{p}) e^{-ip \cdot x} + a^\dagger(\mathbf{p}) e^{ip \cdot x} \right],$$

where  $p^0 = E_p$ .

The operators  $a(\mathbf{p})$  and  $a^\dagger(\mathbf{p})$  satisfy

$$[a(\mathbf{p}), a^\dagger(\mathbf{q})] = (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{q}),$$

$$[a(\mathbf{p}), a(\mathbf{q})] = 0, \quad [a^\dagger(\mathbf{p}), a^\dagger(\mathbf{q})] = 0.$$

The operator  $a^\dagger(\mathbf{p})$  is a creation operator: it creates one particle with momentum  $\mathbf{p}$ . The operator  $a(\mathbf{p})$  is an annihilation operator: it removes one such particle.

The vacuum state  $|0\rangle$  is defined by

$$a(\mathbf{p})|0\rangle = 0 \quad \text{for all } \mathbf{p}.$$

A one-particle state is

$$|\mathbf{p}\rangle = a^\dagger(\mathbf{p})|0\rangle.$$

A two-particle state is

$$|\mathbf{p}, \mathbf{q}\rangle = a^\dagger(\mathbf{p})a^\dagger(\mathbf{q})|0\rangle.$$

Because the creation operators commute,

$$a^\dagger(\mathbf{p})a^\dagger(\mathbf{q}) = a^\dagger(\mathbf{q})a^\dagger(\mathbf{p}),$$

the two-particle state is symmetric under exchange. This is the quantum-field-theoretic origin of Bose-Einstein statistics for scalar particles.

The Hilbert space built by applying creation operators to the vacuum is called Fock space. It contains sectors with zero particles, one particle, two particles, and so on:

$$\mathcal{F} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \dots$$

This is already a major improvement over fixed-particle-number relativistic quantum mechanics. A quantum field theory naturally allows states with variable particle number.

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## 4.4 Number, energy, and normal ordering

Substituting the mode expansion into the Hamiltonian gives

$$H = \int \frac{d^3p}{(2\pi)^3} E_{\mathbf{p}} \left[ a^\dagger(\mathbf{p})a(\mathbf{p}) + \frac{1}{2}(2\pi)^3 \delta^{(3)}(\mathbf{0}) \right].$$

The first term counts real excitations. The second term is the sum of zero-point energies from infinitely many harmonic oscillators. It is formally divergent.

The operator

$$N = \int \frac{d^3p}{(2\pi)^3} a^\dagger(\mathbf{p})a(\mathbf{p})$$

is the number operator. It counts how many scalar particles are present.

For many scattering calculations, one removes the infinite vacuum energy by normal ordering. Normal ordering means placing all creation operators to the left of all annihilation operators. It is denoted by colons:

$$: H := \int \frac{d^3p}{(2\pi)^3} E_{\mathbf{p}} a^\dagger(\mathbf{p})a(\mathbf{p}).$$

Normal ordering is not a magical deletion of physics. It defines the vacuum energy to be zero relative to the chosen free-field vacuum. In nongravitational scattering calculations, only energy differences usually matter. In contexts where gravity couples to vacuum energy, the issue is much deeper and cannot be dismissed by notation alone.

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## 4.5 Complex scalar fields and antiparticles

A real scalar field describes neutral spin-0 particles. To describe charged spin-0 particles, one uses a complex scalar field  $\phi$ , with independent field  $\phi^\dagger$ . The free Lagrangian is

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi.$$

The mode expansion has two independent sets of operators:

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} [a(\mathbf{p})e^{-ip \cdot x} + b^\dagger(\mathbf{p})e^{ip \cdot x}],$$

$$\phi^\dagger(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} [b(\mathbf{p})e^{-ip \cdot x} + a^\dagger(\mathbf{p})e^{ip \cdot x}].$$

Here  $a^\dagger$  creates a particle and  $b^\dagger$  creates an antiparticle. Both have positive energy.

This is the field-theoretic resolution of the negative-energy problem from relativistic wave mechanics. The negative-frequency part of the classical solution does not become a negative-energy particle. After quantization, it becomes the creation operator for a positive-energy antiparticle.

For a charged scalar field, the conserved charge takes the schematic form

$$Q = \int \frac{d^3p}{(2\pi)^3} [a^\dagger(\mathbf{p})a(\mathbf{p}) - b^\dagger(\mathbf{p})b(\mathbf{p})],$$

up to the chosen unit of charge. Particles and antiparticles carry opposite charge.

This was precisely the kind of reinterpretation emphasized in the early field-theoretic treatment of the Klein-Gordon equation by Pauli and Weisskopf (1934).

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## 4.6 Propagators: amplitudes for field disturbances

A central object in quantum field theory is the propagator. Informally, a propagator measures the amplitude for a field excitation created at one spacetime point to be destroyed at another.

For the scalar field, the most important propagator is the Feynman propagator,

$$\Delta_F(x - y) = \langle 0 | T \phi(x) \phi(y) | 0 \rangle.$$

The symbol  $T$  denotes time ordering. It places the operator at the later time to the left:

$$T \phi(x) \phi(y) = \begin{cases} \phi(x) \phi(y), & x^0 > y^0, \\ \phi(y) \phi(x), & y^0 > x^0. \end{cases}$$

The Feynman propagator has the momentum-space representation

$$\Delta_F(x - y) = \int \frac{d^4 p}{(2\pi)^4} \frac{i e^{-i p \cdot (x - y)}}{p^2 - m^2 + i\epsilon}.$$

The small term  $i\epsilon$  tells us how to avoid the poles in the complex  $p^0$ -plane. It encodes the boundary condition appropriate to time-ordered propagation.

The propagator is a Green function for the Klein-Gordon operator:

$$(\square_x + m^2) \Delta_F(x - y) = -i \delta^{(4)}(x - y).$$

This equation says that the propagator is the response of the free quantum field to a pointlike source. Later, internal lines in Feynman diagrams will be propagators.

Example: in electron-muon scattering, the exchanged virtual photon will be represented by a photon propagator. In a scalar theory, the analogous internal scalar line would be represented by  $\Delta(F)$ .

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## 4.7 Microcausality

Relativity forbids signals from traveling faster than light. In quantum field theory this principle appears as microcausality.

Two spacetime points  $x$  and  $y$  are called spacelike separated if

$$(x - y)^2 < 0.$$

For spacelike separation, no light signal can travel from one point to the other. Therefore, measurements localized near  $x$  should not influence measurements localized near  $y$ .

For a real scalar field, the commutator is

$$[\phi(x), \phi(y)] = i\Delta(x - y),$$

where  $\Delta(x-y)$  is the Pauli-Jordan function. A key property is

$$\Delta(x - y) = 0 \quad \text{if } (x - y)^2 < 0.$$

Thus

$$[\phi(x), \phi(y)] = 0 \quad \text{for spacelike separation.}$$

This is microcausality.

It is important not to misunderstand this statement. The Feynman propagator  $\Delta(F)(x-y)$  does not vanish outside the light cone. But the commutator does. The propagator is not by itself a signal. The commutator controls whether a local operation at one point can affect measurement outcomes at another spacelike-separated point.

In other words:

> Quantum amplitudes may have spacelike correlations, but controllable causal influence cannot propagate outside the light cone.

This distinction is essential throughout relativistic quantum theory.

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## 4.8 The Dirac field and the need for anticommutators

The free Dirac Lagrangian is

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi,$$

where

$$\bar{\psi} = \psi^\dagger \gamma^0.$$

The Dirac equation is

$$(i\gamma^\mu \partial_\mu - m)\psi = 0.$$

The field  $\psi$  carries spinor indices, though we often suppress them. Its mode expansion is

$$\psi(x) = \sum_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} [b_s(\mathbf{p})u_s(\mathbf{p})e^{-ip \cdot x} + d_s^\dagger(\mathbf{p})v_s(\mathbf{p})e^{ip \cdot x}].$$

Here:

- $b_s^\dagger(\mathbf{p})$  creates a fermion with spin label  $s$ ,
- $d_s^\dagger(\mathbf{p})$  creates an antifermion,
- $u_s(\mathbf{p})$  is a positive-energy Dirac spinor,
- $v_s(\mathbf{p})$  is a negative-frequency spinor reinterpreted as creating an antiparticle.

For QED,  $b_s^\dagger$  creates an electron and  $d_s^\dagger$  creates a positron.

Unlike scalar fields, Dirac fields are quantized with anticommutators rather than commutators. The anticommutator is

$$\{A, B\} \equiv AB + BA.$$

The equal-time canonical anticommutation relations are

$$\{\psi_\alpha(t, \mathbf{x}), \psi_\beta^\dagger(t, \mathbf{y})\} = \delta_{\alpha\beta} \delta^{(3)}(\mathbf{x} - \mathbf{y}),$$

$$\{\psi_\alpha(t, \mathbf{x}), \psi_\beta(t, \mathbf{y})\} = 0, \quad \{\psi_\alpha^\dagger(t, \mathbf{x}), \psi_\beta^\dagger(t, \mathbf{y})\} = 0.$$

Equivalently, the creation and annihilation operators satisfy

$$\{b_s(\mathbf{p}), b_{s'}^\dagger(\mathbf{q})\} = (2\pi)^3 \delta_{ss'} \delta^{(3)}(\mathbf{p} - \mathbf{q}),$$

$$\{d_s(\mathbf{p}), d_{s'}^\dagger(\mathbf{q})\} = (2\pi)^3 \delta_{ss'} \delta^{(3)}(\mathbf{p} - \mathbf{q}),$$

with all other anticommutators vanishing.

This is not an arbitrary choice. Integer-spin fields are quantized with commutators, while half-integer-spin fields are quantized with anticommutators. This connection between spin and statistics is a structural theorem of relativistic quantum field theory, established in its classic form by Pauli (1940) and treated systematically in modern texts such as Weinberg (1995).

The anticommutation relation has an immediate consequence. Since

$$\{b_s^\dagger(\mathbf{p}), b_s^\dagger(\mathbf{p})\} = 0,$$

we have

$$(b_s^\dagger(\mathbf{p}))^2 = 0.$$

Two identical fermions cannot occupy the same quantum state. This is the Pauli exclusion principle.

Example: two electrons can occupy the same spatial orbital in an atom only if they differ in spin. They cannot have the same complete set of quantum numbers.

After normal ordering, the free Dirac Hamiltonian becomes

$$H = \sum_s \int \frac{d^3p}{(2\pi)^3} E_{\mathbf{p}} [b_s^\dagger(\mathbf{p})b_s(\mathbf{p}) + d_s^\dagger(\mathbf{p})d_s(\mathbf{p})] .$$

Both electrons and positrons have positive energy.

The charge operator is

$$Q = -e \sum_s \int \frac{d^3p}{(2\pi)^3} [b_s^\dagger(\mathbf{p})b_s(\mathbf{p}) - d_s^\dagger(\mathbf{p})d_s(\mathbf{p})] ,$$

where  $e > 0$  is the magnitude of the electron charge. Thus electrons have charge  $-e$ , while positrons have charge  $+e$ .

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## 4.9 The fermion propagator

The Dirac Feynman propagator is

$$S_F(x - y) = \langle 0 | T \psi(x) \bar{\psi}(y) | 0 \rangle .$$

Because  $\psi$  is fermionic, time ordering includes a minus sign when two fermionic operators are exchanged:

$$T \psi(x) \bar{\psi}(y) = \begin{cases} \psi(x) \bar{\psi}(y), & x^0 > y^0, \\ -\bar{\psi}(y) \psi(x), & y^0 > x^0. \end{cases}$$

In momentum space,

$$S_F(x-y) = \int (d^4p) \frac{1}{(2\pi)^4} \frac{\text{slashed } p + m}{i(p^2 - m^2 + i\epsilon)} e^{-ip \cdot (x-y)}$$

where

$$\text{slashed } p \equiv \gamma^\mu p_\mu .$$

This propagator is a Green function for the Dirac operator:

$$(i\gamma^\mu \partial_\mu - m)S_F(x - y) = i\delta^{(4)}(x - y).$$

The numerator slashed  $p+m$  carries spinor information. The denominator is the same relativistic mass-shell structure that appeared for the scalar field.

This pattern will become one of the most useful facts in practical QED:

- scalar propagator:

$$\frac{i}{p^2 - m^2 + i\epsilon}$$

- Dirac propagator:

$$(i(\text{slashed } p+m)) \square (p \square - m \square + i\epsilon).$$

The difference is not the pole location; both describe particles of mass  $m$ . The difference is the spin structure.

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## 4.10 Quantizing the free electromagnetic field

The electromagnetic field is subtler than scalar or Dirac fields because it contains gauge redundancy.

The classical electromagnetic field strength is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and the free Maxwell Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

The potential  $A_\mu$  is not unique. The transformation

$$A_\mu \longrightarrow A_\mu + \partial_\mu \Lambda$$

leaves  $F(\mu\nu)$  unchanged. Therefore  $A_\mu$  contains redundant variables. Quantizing all four components naively would introduce unphysical degrees of freedom.

A photon has only two physical polarization states. For a photon with momentum  $\mathbf{k}$ , the physical polarizations are transverse:

$$\mathbf{k} \cdot \boldsymbol{\epsilon}_\lambda(\mathbf{k}) = 0, \quad \lambda = 1, 2.$$

One direct way to quantize the free photon is to use radiation gauge, also called Coulomb gauge for the free field:

$$A^0 = 0, \quad \nabla \cdot \mathbf{A} = 0.$$

Then only the transverse part of  $\mathbf{A}$  remains dynamical.

The mode expansion is

$$A_i(x) = \sum_{\lambda=1}^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2|\mathbf{k}|}} \left[ \epsilon_i^{(\lambda)}(\mathbf{k}) a_\lambda(\mathbf{k}) e^{-ik \cdot x} + \epsilon_i^{(\lambda)*}(\mathbf{k}) a_\lambda^\dagger(\mathbf{k}) e^{ik \cdot x} \right],$$

where

$$k^0 = |\mathbf{k}|.$$

The photon creation and annihilation operators satisfy

$$[a_\lambda(\mathbf{k}), a_{\lambda'}^\dagger(\mathbf{q})] = (2\pi)^3 \delta_{\lambda\lambda'} \delta^{(3)}(\mathbf{k} - \mathbf{q}),$$

with all other commutators vanishing.

A one-photon state is

$$|\mathbf{k}, \lambda\rangle = a_\lambda^\dagger(\mathbf{k})|0\rangle.$$

The free photon Hamiltonian, after normal ordering, is

$$H_\gamma = \sum_{\lambda=1}^2 \int \frac{d^3k}{(2\pi)^3} |\mathbf{k}| a_\lambda^\dagger(\mathbf{k}) a_\lambda(\mathbf{k}).$$

The photon is massless, so its energy is

$$E_{\mathbf{k}} = |\mathbf{k}|.$$

Because photons are spin-1 massless particles, their physical polarization states can also be labeled by helicity,

$$h = \pm 1,$$

where helicity is the projection of spin along the direction of momentum.

The covariant quantization of the photon requires gauge fixing and, in non-Abelian gauge theories, ghost fields. For QED, ghosts decouple in covariant gauges. We postpone that systematic treatment to Chapter 13. For now, the main point is physical:

> The free electromagnetic field has two independent quantum oscillators per momentum, corresponding to the two physical photon polarizations.

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## 4.11 The photon propagator

In practical QED calculations, it is often convenient to use a covariant gauge rather than radiation gauge. A common choice is Feynman gauge. After gauge fixing, the photon propagator in Feynman gauge is

$$D_{\mu\nu}(x-y) = \langle 0|T A_\mu(x) A_\nu(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4} \frac{-i\eta_{\mu\nu} e^{-ip\cdot(x-y)}}{p^2 + i\epsilon}.$$

More generally, in a covariant Rxi-type gauge,

$$D_{\mu\nu}^{(\xi)}(p) = \frac{-i}{p^2 + i\epsilon} \left[ \eta_{\mu\nu} - (1 - \xi) \frac{p_\mu p_\nu}{p^2 + i\epsilon} \right].$$

The parameter  $\xi$  labels the gauge choice. Physical observables cannot depend on  $\xi$ . This gauge independence will later be enforced by Ward identities.

There is an important lesson here. The photon propagator may contain unphysical gauge-dependent pieces, but measurable scattering amplitudes do not. Gauge redundancy is allowed in intermediate calculations only because it cancels out of final physical predictions.

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## 4.12 Microcausality for spinors and photons

For the scalar field, microcausality was expressed by a vanishing commutator at spacelike separation. For fermions, the field itself is not an observable; it changes fermion number and carries spinor indices. The appropriate locality condition is that fermion fields anticommute at spacelike separation in such a way that local observables commute.

For the Dirac field,

$$\{\psi_\alpha(x), \bar{\psi}_\beta(y)\}$$

vanishes outside the light cone up to the standard Dirac operator acting on the scalar causal function. Consequently, observables such as

$$\bar{\psi}(x)\psi(x), \quad \bar{\psi}(x)\gamma^\mu\psi(x)$$

commute with corresponding local observables at spacelike separation.

For the electromagnetic field, the gauge potential  $A_\mu$  is not itself gauge invariant. The physical field strength  $F_{\mu\nu}$  is gauge invariant, and local commutators of electromagnetic field strengths vanish at spacelike separation.

Thus microcausality is best stated as:

> Gauge-invariant local observables commute at spacelike separation.

This is the form of locality that matters physically.

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## 4.13 What is a particle?

In nonrelativistic quantum mechanics, a particle is usually taken as primitive. In quantum field theory, particles are derived objects.

For a free field, a particle is a one-quantum excitation of a normal mode. The state

$$a^\dagger(\mathbf{p})|0\rangle$$

has definite momentum and energy. It lies on the relativistic mass shell,

$$p^2 = m^2, \quad p^0 > 0.$$

For the Dirac field, the one-particle states are

$$b_s^\dagger(\mathbf{p})|0\rangle$$

for electrons and

$$d_s^\dagger(\mathbf{p})|0\rangle$$

for positrons. For the electromagnetic field, the one-particle states are

$$a_\lambda^\dagger(\mathbf{k})|0\rangle$$

for photons.

This definition is cleanest in a free theory. In an interacting theory, particles can be emitted, absorbed, and dressed by clouds of virtual quanta. The electron observed in a detector is not merely the bare field  $\psi$ ; it is an asymptotic charged excitation whose precise description becomes subtle, especially because photons are massless. These infrared subtleties will matter in Chapter 17.

Nevertheless, free-particle states remain the starting point of scattering theory. We prepare incoming particles far in the past, let them interact, and measure outgoing particles far in the future. The bridge between those asymptotic states is the S-matrix, which we begin constructing in Chapter 8.

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## 4.14 The hierarchy of free QED fields

Before introducing interactions, it is useful to place the three free fields side by side.

The scalar field is pedagogically simplest:

$$\phi \longrightarrow \text{spin-0 bosons.}$$

The Dirac field is the matter field of QED:

$$\psi \longrightarrow \text{electrons and positrons.}$$

The electromagnetic field is the gauge field of QED:

$$A_\mu \longrightarrow \text{photons.}$$

Their basic quantization rules are different:

bosons: commutators,

fermions: anticommutators.

Their propagators are different:

$$\Delta_F(p) = \frac{i}{p^2 - m^2 + i\epsilon},$$

$$S_F(p) = (i(\text{slashed } p + m)) \square (p \square - m \square + i\epsilon),$$

$$D_{\mu\nu}(p) = \frac{-i\eta_{\mu\nu}}{p^2 + i\epsilon} \quad \text{in Feynman gauge.}$$

But the logic is unified. In every case:

1. Start from a classical free Lagrangian.

2. Identify canonical variables.
3. Promote fields to operators.
4. Impose commutation or anticommutation relations.
5. Build Fock space from creation operators.
6. Identify propagators as vacuum expectation values of time-ordered fields.
7. Enforce locality through microcausality.

This is the free-field foundation of QED.

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## **4.15 Chapter summary**

A quantum field is an operator-valued distribution

# Document information

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